Operator Theory

Assignment 1— to be submitted Tuesday Aban 18, 1395 .

Instructor: S. M. Manjegani

1. Let $\{T_i\}$ be a bounded net in $B(H)$ and fix orthogonormal basis$\{e_n\}$.
   (a) Prove that $T_i \longrightarrow 0$ (WOT) if and only if $\langle T_i e_n, e_m\rangle \longrightarrow 0$ for all e_n and e_m.
   (b) Prove that $T_i \longrightarrow 0$ (SOT) if and only if for all e_n, $\|T_i e_n\| \longrightarrow 0$

2. Let $\{T_i\}$ and $\{S_i\}$ be nets in $B(H)$ and assume that $\{T_i\}$ is uniformly bounded.
   (a) If $T_i \longrightarrow 0$ (WOT) and $S_i \longrightarrow 0$ (SOT), then $T_i S_i \longrightarrow 0$ (WOT).
   (b) If, in addition, $T_i \longrightarrow 0$ (SOT), then $T_i S_i \longrightarrow 0$ (WOT).

3. Let $B_{00}(H)$ denote the algebra of finite rank on Hilbert space H. Show that ball$B_{00}(H)$ is WOT (respectively, SOT) dense in ball$B(H)$.

4. If $T : H \longrightarrow K$ is acompact operator and $\{e_h\}$ is any orthogonal sequence in H, then $\|Te_n\| \longrightarrow 0$. Is the coverse true?

5. Let T be a compact operator. Show that either $\|T\|$ or $-\|T\|$ is an eigenvalue of T.

6. If T is a compact operator on a Hilbert space H, then

$$T = \sum_{n=1}^{\infty} \lambda_n P_n \,,$$

   where $\{\lambda_n\}$ is the set of eigenvalues of T and P_n is the projection of H onto $\ker(T - \lambda_n)$. The series converges to T in norm.